Factoring a polynomial is the process of writing it as the product of two or more polynomial factors.

**Example:** $7x^2 + 35x + 42 = 7(x + 2)(x + 3)$— one monomial factor (7) and two binomial factors $(x + 2)$ and $(x + 3)$

Set the factors of a polynomial equation (as opposed to an expression) equal to zero in order to solve for a variable: **Example:** To solve $7x^2 + 35x + 42 = 0 \rightarrow x + 2 = 0, x = -2$; and $x + 3 = 0, x = -3$

The flowchart below illustrates a sequence of steps for factoring polynomials.

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**First, always factor out the Greatest Common Factor (GCF), if one exists.**

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**Is the equation a Binomial or a Trinomial?**

- **No**
  - **Four or more terms**
  - Factor by Grouping:
    1. Group the terms with common factors and factor out the GCF from each grouping.
    2. Continue factoring—by looking for Special Cases, Grouping, etc.—until the equation is in simplest form (or all factors are Prime).

- **Yes**
  - **Special Cases?**
    - **Binomial (two terms)**
      - **Perfect Square Trinomial:**
        1. $x^2 + 2xy + y^2 = (x + y)^2$
        2. $x^2 - 2xy + y^2 = (x - y)^2$
    - **Trinomial (three terms)**
      - **Perfect Square Trinomial:**
        1. $x^2 + 2xy + y^2 = (x + y)^2$
        2. $x^2 - 2xy + y^2 = (x - y)^2$

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**No Special Cases**

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**Choose:**

1. Factor by Grouping
2. Complete the Square
3. Use the Quadratic Formula

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This process is applied in the following examples

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Factoring steps and most examples are adapted from Professor Elias Juridini, Lamar State College-Orange.
**Factoring Examples**

## Binomials:

1. **6x^2 + 12x**: First, divide each term by the GCF to get the quotient \( \frac{6x^2}{6x} + \frac{12x}{6x} = x + 2 \).

   Then, show the quotient multiplied by the GCF \( 6x(x + 2) \).

   No special cases apply to the binomial quotient \( x + 2 \), so the factors are \( 6x \) and \( x + 2 \).

   **Factored:** \( 6x^2 + 12x = 6x(x + 2) \)

2. **x^4 – 16**: This polynomial expression has no GCF (*other than 1 and –1*).

   However, it can be expressed as a **Difference of Squares**: \( a^2 – b^2 \rightarrow (x^2)^2 – 4^2 \).

   Use the square root of each term, \( \sqrt{x^4} = \sqrt{(x^2)^2} = x^2 \) and \( \sqrt{16} = \sqrt{4^2} = 4 \) to fill in the formula:

   *If* \( a = x^2 \) and \( b = 4 \), then ...

   \[ a^2 – b^2 = (a – b)(a + b) \]

   \[ (x^2)^2 – 4^2 = (x^2 – 4)(x^2 + 4) \]

   Continue to factor another **Difference of Squares**: \( x^2 – 4 = (x – 2)(x + 2) \).

   The next factor \( x^2 + 4 \) is a **Sum of Squares**, which is **Prime**.

   **Factored:** \( x^4 – 16 = (x – 2)(x + 2)(x^2 + 4) \)

3. **729x^3 – 1**: There’s no GCF to factor out, but you should recognize this polynomial expression as a **Difference of Cubes**: \( a^3 – b^3 \rightarrow 9^3x^3 – 1^3 \).

   Use the cube root of each term, \( \sqrt[3]{729x^3} = \sqrt[3]{9^3x^3} = 9x \) and \( \sqrt[3]{1} = \sqrt[3]{1^3} = 1 \) to fill in the formula:

   \[ a^3 – b^3 = (a – b)(a^2 + ab + b^2) \]

   *In the first set of parentheses, place each term to the first power: \( (9x – 1) \).*

   *In the second set, square the first and the last term: \( [(9x)^2 + ? + 1^2] \)

   *The middle term is the product of the first and the last: \( [(9x)^2 + 9x(1) + 1^2] \)

   *Note that this quadratic factor is Prime!*

   *Use S.O.A.P to remember the signs: Same in first set of parentheses; Opposite, followed by Always Positive in second set of parentheses.*

   If \( a = 9x \) and \( b = 1 \), then ...

   \[ a^3 – b^3 = (a – b)(a^2 + ab + b^2) \]

   \[ 9^3x^3 – 1^3 = (9x – 1)[9^2x^2 + 9x(1) + 1^2] \]

   **Factored:** \( 729x^3 – 1 = (9x – 1)(81x^2 + 9x + 1) \)

## Trinomials:

1. **x^2 + 4x + 4**: There’s no GCF to factor out of this expression, so check for a **Perfect Square Trinomial**—

   \[ x^2 + 2xy + y^2 = (x + y)^2 \text{ or } x^2 – 2xy + y^2 = (x – y)^2 \]

   **ASK:** *Are the first and the last terms perfect squares? \( x^2 \) and \( 4 = 2^2 \) — **Yes!**

   *Is the middle term two times the product of the square roots of the first and last terms?*

Factoring steps and most examples are adapted from Professor Elias Juridini, Lamar State College-Orange.
First Term \( x^2 \rightarrow \sqrt{x^2} = x \), Last Term \( 4 \rightarrow \sqrt{4} = 2 \), and \( 2(2x) = 4x \) — Yes!

If “Yes” to both of the above:

- Place the square roots of the first and last terms in parentheses: \((x \ ? \ 2)\)
- Use the sign of the middle term: \((x + 2)\)
- Square the whole thing: \((x + 2)^2\)

If \( \sqrt{x^2} = x \) and \( \sqrt{4} = 2 \), then ...

\[
x^2 + 2xy + y^2 = (x + y)^2
\]

\[
x^2 + 4x + 4 \rightarrow x^2 + 2(2x) + 2^2 = (x + 2)^2
\]

**Factored:** \( x^2 + 4x + 4 = (x + 2)^2 \)

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2. \( 4x^2 + 2x - 20 = 0 \): Factor out the GFC, 2, in this equation to get \( 2(2x^2 + x - 10) \).

- **Grouping:**
  - First, multiply the coefficient of the first term, 2, and the constant, 10: \( 2 \cdot 10 = 20 \).
  - Then find the factors of this product, 20, that add/subtract to yield the coefficient of the middle term, which is 1.
  - Factors of 20: \( 1 \cdot 20 \), \( 2 \cdot 10 \), and \( 4 \cdot 5 \). Of these factors, 4 and 5 will subtract to give 1. Because the middle term is positive, we use +5 and -4.
  - Replace the middle term with -4 and +5: \( 2x^2 - 4x + 5x - 10 \).
    - It’s important to order the two new middle terms so a GCF can be factored from each pair: \( 2x^2 - 4x + 5x - 10 \), but not \( 2x^2 + 5x - 4x - 10 \).
  - Group the first and last pairs of terms; factor out a GCF from each, and rewrite the problem: \((2x^2 - 4x) \text{ and } (5x - 10) \rightarrow 2x(x - 2) \text{ and } 5(x - 2) \rightarrow 2x(x - 2) + 5(x - 2)\).
    - Note that \((x - 2)\) appears twice. If this doesn’t happen, reorder your middle terms.
  - Factor out the GCF—which is \((x - 2)\)—and rewrite in factored form.
    - \( 2x(x - 2) + 5(x - 2) \rightarrow (x - 2)(2x + 5) \)
  - Remember to include the 2 factored out at the beginning when you write the whole equation in factored form

**Factored:** \( 4x^2 + 2x - 20 = 0 \rightarrow 2(x - 2)(2x + 5) = 0 \)

- Recall that this example is an equation set equal to zero (not simply an expression). That means we can solve for \( x \) by setting each factor containing a variable equal to zero.

\[
x - 2 = 0 \text{ or } 2x + 5 = 0
\]

\[
x - 2 + 2 = 0 + 2 = 2 \text{ or } 2x + 5 - 5 = 0 - 5 \rightarrow \frac{2x}{2} = -\frac{5}{2}
\]

**Solutions:** \( x = 2 \) or \( x = -\frac{5}{2} \)

*Always check by plugging your answers into the original equation to verify them!

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b. Now let’s solve the same equation, \( 4x^2 + 2x - 20 = 0 \), by Completing the Square:

- Make sure the equation is in the General (quadratic) Form—\( ax^2 + bx + c = 0 \)—and it is.
- The GCF is 2, but you don’t need to factor it out—doing so will not affect the solutions.
Factoring steps and most examples are adapted from Professor Elias Juridini, Lamar State College-Orange.
• Note that factoring out the 2 will not affect the solutions, but it does result in smaller coefficients to plug into the Quadratic Formula.
• Make sure the equation is in General (quadratic) Form, \( ax^2 + bx + c = 0 \), and it is.
• Identify \( a, b \) and \( c \) in \( 2x^2 + x - 10 = 0 \): \( a = 2, \ b = 1, \ c = -10 \)
• Plug these numbers into the formula and solve for \( x \):

\[
x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-10)}}{2(2)}
\]

\[
x = \frac{-1 \pm \sqrt{1 + 80}}{4} \rightarrow \frac{-1 \pm \sqrt{81}}{4} \rightarrow \frac{-1 \pm 9}{4} \rightarrow \frac{-1 + 9}{4} = \frac{8}{4} = \frac{2}{1} \text{ or } \frac{-1 - 9}{4} = -\frac{10}{4} = -\frac{5}{2}
\]

Solutions: \( x = 2 \) or \( x = -\frac{5}{2} \)

*Again, we obtain the same solutions as we did by other methods!*

More Than Four Terms:

1. \( 5x^2 - 60y - 10y^2 + 10x \): Factor the expression by Grouping and continue factoring until completely simplified.
   • Factor out the GCF, 5, to get \( 5(x^2 - 12y - 2y^2 + 2x) \) and then group the terms so that another GCF can be removed from each grouping.

\[
5(x^2 + 2x - 2y^2 - 12y) \rightarrow 5[x(x + 2) - 2y(y + 6)]
\]

• We are unable to obtain a pair of identical binomial factors (as in the earlier Grouping example). Therefore, no further factoring is possible and we see that the expression has only two factors, 5 and \( [x(x + 2) - 2y(y + 6)] \).

Factored: \( 5x^2 - 60y - 10y^2 + 10x = 5[x(x + 2) - 2y(y + 6)] \)

2. \( x^2 - y^2 + 6x + 9 \): The terms of this expression appear to need no rearrangement. The first pair, \( x^2 - y^2 \), are a Difference of Squares and the second pair, \( 6x + 9 \), have a GCF of 3:
   • Factor each pair: \( x^2 - y^2 + 6x + 9 = (x - y)(x + y) + 3(2x + 3) \)

However, there is no GCF to factor out of these pairs of factors. The expression remains the sum of polynomial products rather than the product of two or more polynomial factors (the definition of the factored form).

• When one way of grouping doesn’t work, try another . . .

Rearrange the terms to form a Perfect Square Trinomial and a Difference of Squares:

\[
x^2 - y^2 + 6x + 9 = x^2 + 6x + 9 - y^2 = (x + 3)^2 - y^2
\]

• Factor the Difference of Squares:

\[
(x + 3)^2 - y^2 = [(x + 3) - y][(x + 3) + y] = (x + 3 - y)(x + 3 + y)
\]

Factored: \( x^2 - y^2 + 6x + 9 = (x + 3 - y)(x + 3 + y) \)